NUMERICAL STUDY OF THERMAL FIELDS IN HIGH-SPEED ELECTRICAL HEATING OF POWDERED MATERIALS

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Three calculation methods are considered for analyzing the temperature fields excited by an electrical current passing through a powdered medium.

Heating of powdered materials by direct passage of electrical current accelerates sintering, permits production of materials with new properties, and reduces energy expenditures for the process [1]. However, wide use of electrical heating has been restrained by insufficient knowledge of the physics of the phenomenon and absence of sound recommendations for selection of optimal regimes. A question of primary importance is reducing nonuniformity of the heating of the powder mixture over volume and time [2, 3].

The rapid propagation of the temperature field in an electrically conductive powder is caused by intrinsic heat liberation, which is dependent on temperature. Such problems are nonlinear. The most suitable calculation methods are those with finite-element discretization over space and finite difference over time [4]. The finite-element discretization of the steady-state field was described in [3].

The entire system (Fig. 1a) can naturally be divided into eight elements. With consideration of symmetry it is sufficient to retain four elements (Fig. 1b). The homogeneous thermally nonconductive cylindrical matrix ensures one-dimensional heat propagation along the cylinder axis OX in both directions from the sintering chamber. To decrease the temperature gradient, segments of the Poisson electrodes with increased electrical resistance are introduced – heaters (Fig. 1a). This levels the temperature field over the powder mixture volume. An explosion-like rise in the powder temperature, caused by a significant decrease in its electrical resistance, can be prevented by a special electrical circuit breaker, which realizes an alternating (two-stage) heating process. The first stage consists of system heating by intrinsic heat liberation in the powder and heaters, and the second, heat dissipation smoothing the time peaks of the temperature.

Solution of the nonsteady-state problem involves minimization (for each moment of the time interval) of the following functional:

$$\chi = \int_V \frac{1}{2}\left[K_x\left(\frac{\partial T}{\partial t}\right)^2 - \left(2Q - 2c\rho\frac{\partial T}{\partial t}\right)T\right]dV + \int_{S_1} qT\,dS + \int_{S_2}\frac{h}{2}(T - T_m)^2\,dS. \quad (1)$$

This reduces Eq. (1) to a system

$$[C]\frac{\partial\{T\}}{\partial t} + [K]\{T\} = \{F\}, \quad (2)$$

where the thermal conductivity matrix [K] and the thermal load vector {F} are analogous [3]. The thermal conductivity matrix of the element is determined by the sum of volume and surface integrals:

$$[k^{(e)}] = \int_{V^{(e)}} K_x[B]^t[B]\,dV + \int_{S_2^{(e)}} h[\Phi]^t[\Phi]\,dS, \quad (3)$$

while the thermal load vector has the form

$$\{f^{(e)}\} = \int_{V^{(e)}} Q[\Phi]^t dV - \int_{S_2^{(e)}} hT_m[\Phi]^t dS. \quad (4)$$

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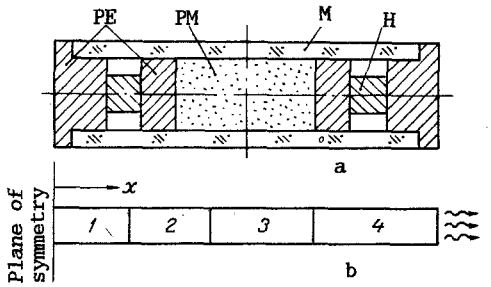

Fig. 1. Press-form: a) schematic diagram: PE) Poisson electrodes; PM) powdered mixtures; M) matrix; H) heater; b) calculation scheme for four finite elements.

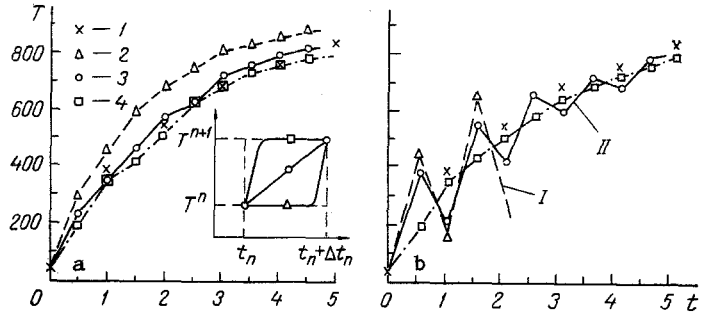

Fig. 2. Change in mean sintering temperature vs time step (a, Δt = 0.25 sec; b, 0.5 sec): 1) experiment [2]; 2) calculation by explicit method (forward difference); 3) Krank—Nicholson method; 4) implicit method (backward difference); I) unstable solution; II) nonphysical oscillations. t, sec.

The solution of the steady-state problem leads to a new matrix [C], called the damping (heat capacity) matrix. For an individual element

$$[c^{(e)}] = \int_{V^{(e)}} c\rho [\Phi]^{\mathrm{t}} [\Phi] \, dV.$$

The boundary conditions of the problem are taken as follows: in the center of the sintering chamber x = 0 the temperature gradient is equal to zero (a natural Neimann condition); on the face heat exchange occurs with the surrounding medium by a Newtonian law, with the heat exchange coefficient h being established empirically. In the calculations the value of h was varied over a wide range. The heat liberation produced by passage of the electrical current in the first and third elements (Fig. 1b) is modeled by the volume integrals in Eq. (4). A special subroutine is provided which considers the temperature dependence of Q. Similar subroutines are written for K_x and c. It is assumed that the parameters ρ and h are temperature-independent. Heat exchange is possible only through the faces of the system, so that the second term in Eqs. (3) and (4) will be nonzero only in the fourth element. For preliminary (qualitative) analysis of the computation scheme simple linear elements are used. Refinement of temperature profiles is achieved by use of quadratic elements.

System (2) can be solved by an entire family [4] of computation schemes:

$$\left\{\frac{1}{\Delta t_n}[C] + \theta[K]\right\}\{T\}^{n+1} + \left\{-\frac{1}{\Delta t_n}[C] + (1-\theta)[K]\right\}\{T\}^n = (1-\theta)\{F\}^n + \theta\{F\}^{n+1}, \qquad (5)$$

where n = 0, 1, 2, ... is the number of the time step. With the aid of a suitable parameter θ from Eq. (5) we obtain a concrete scheme for Eq. (2). The experimental data of [2] were compared with results of calculations by the three best known finite-difference methods (Fig. 2).

In the explicit method (θ = 0) it is assumed that at time t_n the temperature takes on the value $\{T\}^n$, which is maintained over the entire time step Δt_n and only at the end of the

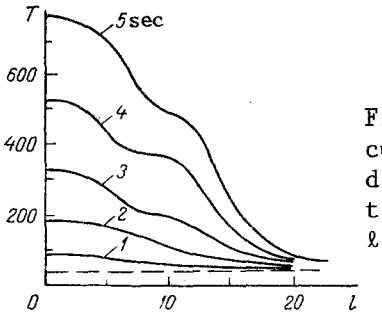

Fig. 3. Temperature–distance curves (dashed line, initial distribution) for various times (digits along curves). ℓ, mm.

interval $t_n + \Delta t_n$ changes abruptly to $\{T\}^{n+1}$ (Fig. 2a). On the other hand, in the implicit method ($\theta = 1$) at time t_n the temperature changes abruptly from $\{T\}^n$ to $\{T\}^{n+1}$ and then remains equal to $\{T\}^{n+1}$ for the entire time step. The Krank–Nicholson method ($\theta = 0.5$) reflects a linear temperature change. Even for constant thermal conductivity and specific heat coefficients the time step of the explicit method is limited by the condition $\Delta t_n < c\rho(\Delta x)^2/2K_x$, violation of which can lead to unstable solutions. The Krank–Nicholson method is usually considered unconditionally stable. However, in this case also oscillations of the solution cannot be excluded (Fig. 2b). In the present calculations results closest to experiment were obtained with the implicit method. The equations of system (2) were solved iteratively with multiply corrected coefficients. The temperature-dependent parameters K_x, c, and Q were recalculated with special subroutines for $\{T\}^{n+1}$.

Analysis of the numerical results allows choice of an optimal time step for each concrete case. This is a practical problem, since the discrete model is quite sensitive to the ratio between time and spatial steps. The experimentally established [2] avalancelike character of the powder heating process is confirmed by the calculations. Heating of the powder mixture is practically independent of the intensity of heat removal from the system faces, since the process lasts only 4-5 sec. This period is insufficient for the temperature wave to reach the face section. Typical curves interpolated for three points of each element for various times are shown in Fig. 3.

NOTATION

$T(x, t)$, temperature of cross section x at time t; K_x, internal thermal conductivity coefficient; Q, heat source intensity; ρ, material density; c, specific heat; V, volume; q, thermal flux on boundary S_1; h, heat exchange coefficient on boundary S_2; T_m, temperature of surrounding medium; $[\Phi]$, form function matrix for finite element; [B], form function derivative matrix; [K], system thermal conductivity matrix; [C], specific heat matrix; t, matrix transposition operation; Δt_n, time step; Δx, step along axis OX.

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